Written Exam for the B.Sc. in Economics 2010-I

# Micro 3

Final Exam

February 2010

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

PLEASE ANSWER ALL QUESTIONS BELOW. PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Find *all* Nash equilibria in the following game

| | L | R |
|---|---|---|
| T | $1,1$ | $3,2$ |
| B | $4,3$ | $2,0$ |

(b) Solve the following game by iterated elimination of strictly dominated strategies

| | $t_1$ | $t_2$ | $t_3$ |
|---|---|---|---|
| $s_1$ | $0,2$ | $1,1$ | $0,2$ |
| $s_2$ | $1,3$ | $1,-1$ | $4,1$ |
| $s_3$ | $2,1$ | $3,1$ | $2,0$ |

(c) Consider the extensive-form game represented by the game tree on Figure 1:

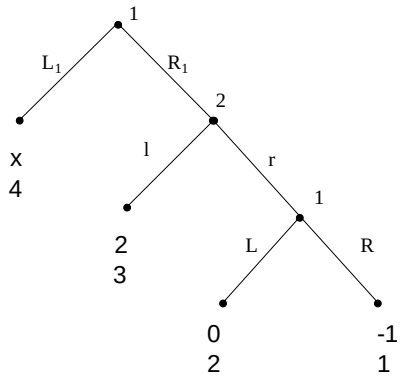


Figure 1

i. Are there any values of $x$ under which the strategy profile $(R_1L, l)$ is a SPNE of this game? If yes, find the respective range of $x$, if not, explain why.

ii. Are there any values of $x$ under which the strategy profile $(L_1L, r)$ is a SPNE of this game? If yes, find the respective range of $x$, if not, explain why. Under which values of $x$ is $(L_1L, r)$ a NE of this game? Comment.

(d) Can a weakly dominated strategy be part of NE? If yes, suggest an example. If no, explain why not. (Be short and precise).

2. Consider the following game: there is a Criminal and a Sheriff. The Criminal selects the seriousness of a crime she is willing to commit, $x > 0$. The Sheriff selects the level of effort he is willing to put into catching the Criminal, $y > 0$. They make these choices simultaneously and non-cooperatively. The utility of the Criminal is given by

$$U_c = (1 - xy)x$$

where $(1 - xy)$ can be interpreted as the probability that the Criminal avoids capture, and $x$ can be interpreted as the value of crime for the Criminal. The utility of the Sheriff is given by

$$U_s = xy - cy^2,$$

where $xy$ represents the probability of catching the criminal, and $cy^2$ is the cost of effort for the Sheriff, $c > 0$.

(a) Assume the effort cost of the Sheriff, represented by parameter $c$, is common knowledge. What are the seriousness of crime $x^*$ and Sheriff's effort level $y^*$ in the Nash equilibrium of this game? In particular, what are they if $c = 1$? If $c = 4$? Explain intuitively how the equilibrium values of $x$ and $y$ change as $c$ increases.

(b) Assume now that there could be two types of Sheriff: lazy, with $c_L = 4$, and hard-working, with $c_H = 1$. The Sheriff knows his own type, but the Criminal does not know which type of Sheriff she is facing. She only knows that the Sheriff can be lazy with probability 2/3 and hard-working with probability 1/3.

i. What are the seriousness of crime $x^{**}$, and the effort levels of the lazy/hard-working types of the Sheriff $y_L^{**}$, $y_H^{**}$ in the Bayes-Nash equilibrium of this game?

ii. The Sheriff knows his type both in incomplete information case of (b) and in complete information case of (a). Does the lazy type of the Sheriff exert the same effort in both cases? Why? Explain intuitively.

3. Consider the following game between Players 1 and 2

| | | Player 2 | |
|---|---|---|---|
| | | X | Y |
| Player 1 | X | $3, 3$ | $0, 4$ |
| | Y | $4, 0$ | $1, 1$ |

(a) Assume this game is repeated 3 times and the payoff of the resulting game is the sum of the payoffs in all three repetitions. Assume there is no time discounting. Is there a SPNE of this game, in which the payoff of either player is equal to 5? Explain your answer.

Assume now that the game is repeated infinitely many times, and each player maximizes net present value of all future discounted payoffs. They both have the discount factor $\delta$, where $0 < \delta < 1$.

(b) Find a range of $\delta$ such that the following strategy is supported as a subgame-perfect equilibrium of this infinitely repeated game:

- Normal phase: play $X$ in the first period of the game or if the play in all past periods was $(X, X)$. Otherwise revert to punishment phase forever.
- Punishment phase: Play Nash equilibrium of the stage game.

(c) Find a range of $\delta$ such that the following strategy is supported as a subgame-perfect equilibrium of this infinitely repeated game.

- Normal phase: play $X$ in the first period of the game or if the play in the past period was $(X, X)$ or $(Y, Y)$. Otherwise revert to punishment phase.
- Punishment phase: Play Nash equilibrium strategy of the stage game *for 1 period*, and revert back to normal phase.

Comment on the intuition behind the difference of your answers in (b) and (c).

4. Three flatmates, Andreas, Bente and Carl, are planning a joint party and discuss how to divide the costs of it. Each of them has made a list of his/her guests, and there are 5 guests in Andreas' list, 7 guests in Bente's list and 5 guests in Carl's list. However, their guest lists partially overlap: there are 8 guests in the joint list of Andreas and Bente, 9 guests in the joint list of Andreas and Carl, and 8 guests in the joint list of Bente and Carl. The list of all guests to the party comprises 11 persons. They estimate the cost per guest to be DKK 100.

(a) Think of this situation as of cooperative game and write down the values of all coalitions.

(b) Assume that the flatmates decided to use the knowledge they have acquired in Micro 3 course and came up with an idea of sharing the costs according to Shapley value. How much will each of them pay?